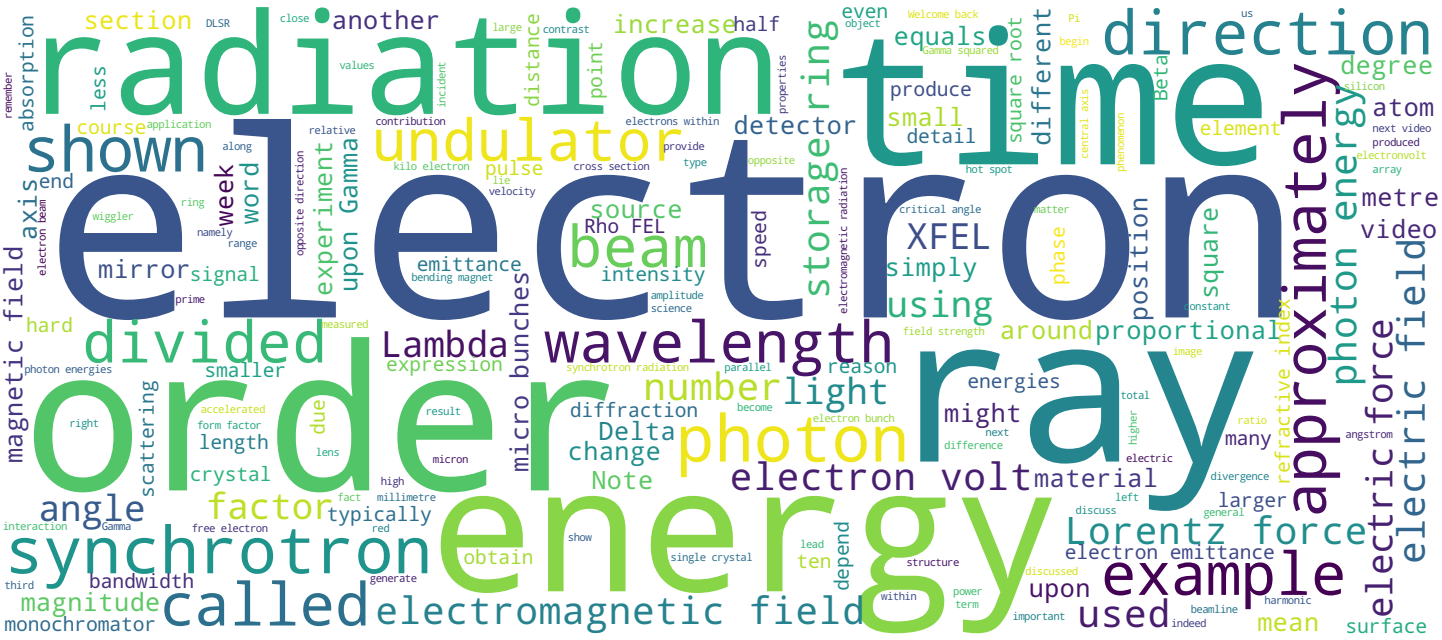
X-ray Free-electron Lasers
(Part 3)
Synchrotrons and x-ray free-electron lasers
Techniques and applications
Prof. Philip Willmott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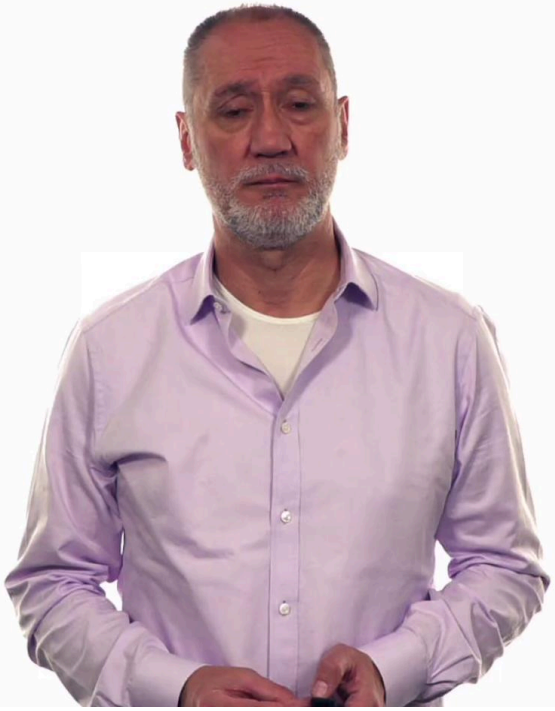
Video

Contents and objectives of this video



- EM forces on electrons
- Influence of a slalom electron path
- Microbunching
- Pierce parameter and gain length
- Seeding and monochromatization

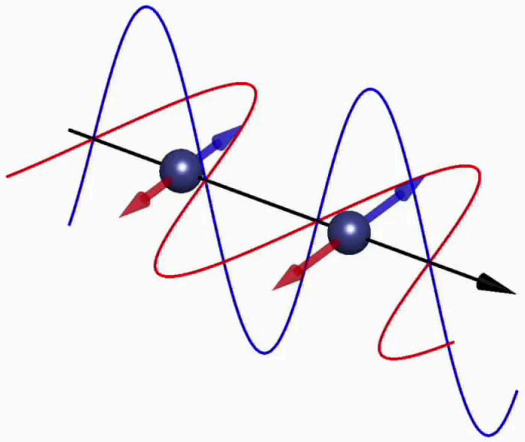
Hi and welcome back to the third and final section of week four at this course, Synchrotron and X-ray Free Electron Laser techniques and applications. In this section we will look in detail at the phenomenon of self-amplified spontaneous emission or SASE, and why this leads to such high intensity, femtosecond duration X-ray pulses. The possibility of generating narrow bandwidth XFEL pulses is also briefly discussed.

Notes

Summary

0m 05s

Pro memoria: EM wave acting on an electron



$$F_E = -eE$$

$$F_L = eBv$$

$$B = \frac{E}{c}$$

$$\Big\} \; F_L/F_E = -v/c$$

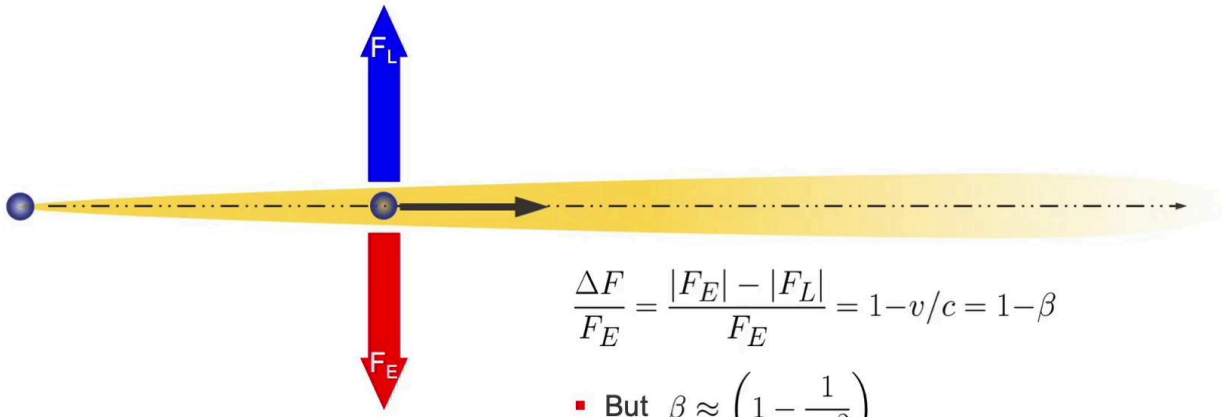
Let's briefly return to a topic we discussed in week three, namely, the forces acting on an electron by a plain wave electromagnetic field. Remember that the force of the electric field component acts in the opposite direction to the field strength due to the negative charge of the electron. If the electromagnetic field and electron move along the same axis, the magnetic field component acts at 90 degrees to its direction and the direction of the electrons motion, resulting in it having a Lorentz force that act in the opposite direction to that of the electric field. Now, the ratio of the Lorentz and electric forces is minus V upon C. Thus, only for relativistic velocities will the magnetic Lorentz force be comparable to the electric force.

Notes

Summary

0m 37s

Forces acting on an electron by EM radiation



$$\frac{\Delta F}{F_E} = \frac{|F_E| - |F_L|}{F_E} = 1 - v/c = 1 - \beta$$

- But $\beta \approx \left(1 - \frac{1}{2\gamma^2}\right)$

$$\Rightarrow \frac{\Delta F}{F_E} = \frac{1}{2\gamma^2} \sim 10^{-8}$$

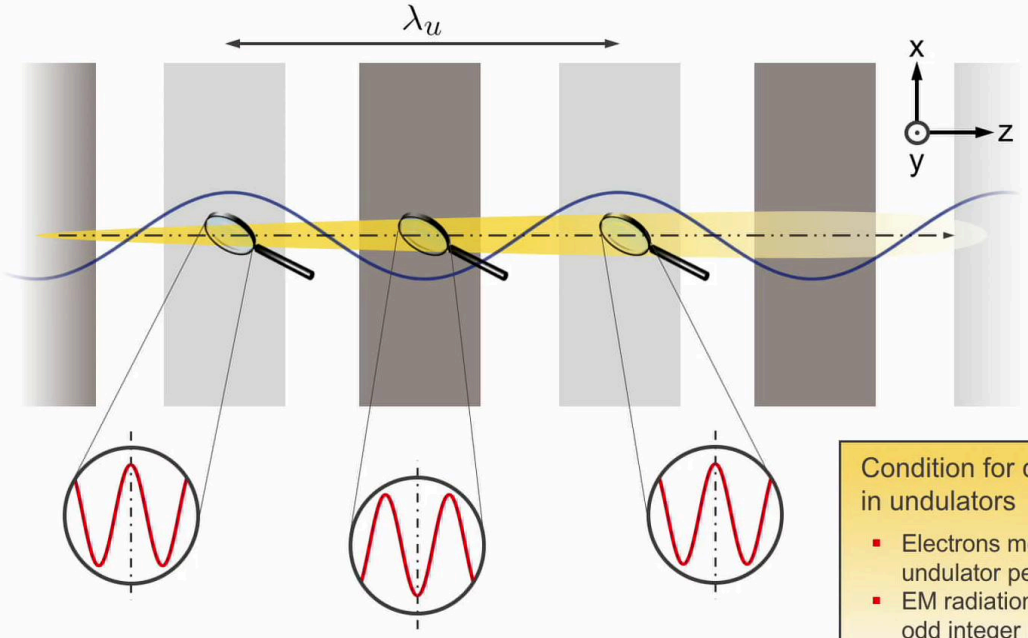
Imagine now light emitted by a first electron along the axis of motion of a second electron. For a given a velocity, V, what is then, the relative difference in the two forces if the electron is co-propagating with the electromagnetic wave? From the forces ratio of minus V upon C, we immediately see that the relative difference is one minus V upon C, which is equal to one minus Beta. But we remember from week three that to a very high degree of accuracy, Beta is equal to one minus one upon two Gamma squared, and hence the relative imbalance in the two forces imparted by an electromagnetic field is simply one divided by two Gamma squared, that is of the order of 10 to the minus eight for an electron in a three giga-electron volt storage ring. So from this we can conclude that the Lorentz force FL, and the electric force FE essentially perfectly cancel one another out.

Notes

Summary

1m 38s

Relationship between λ_u and EM radiation



Condition for constructive interference in undulators

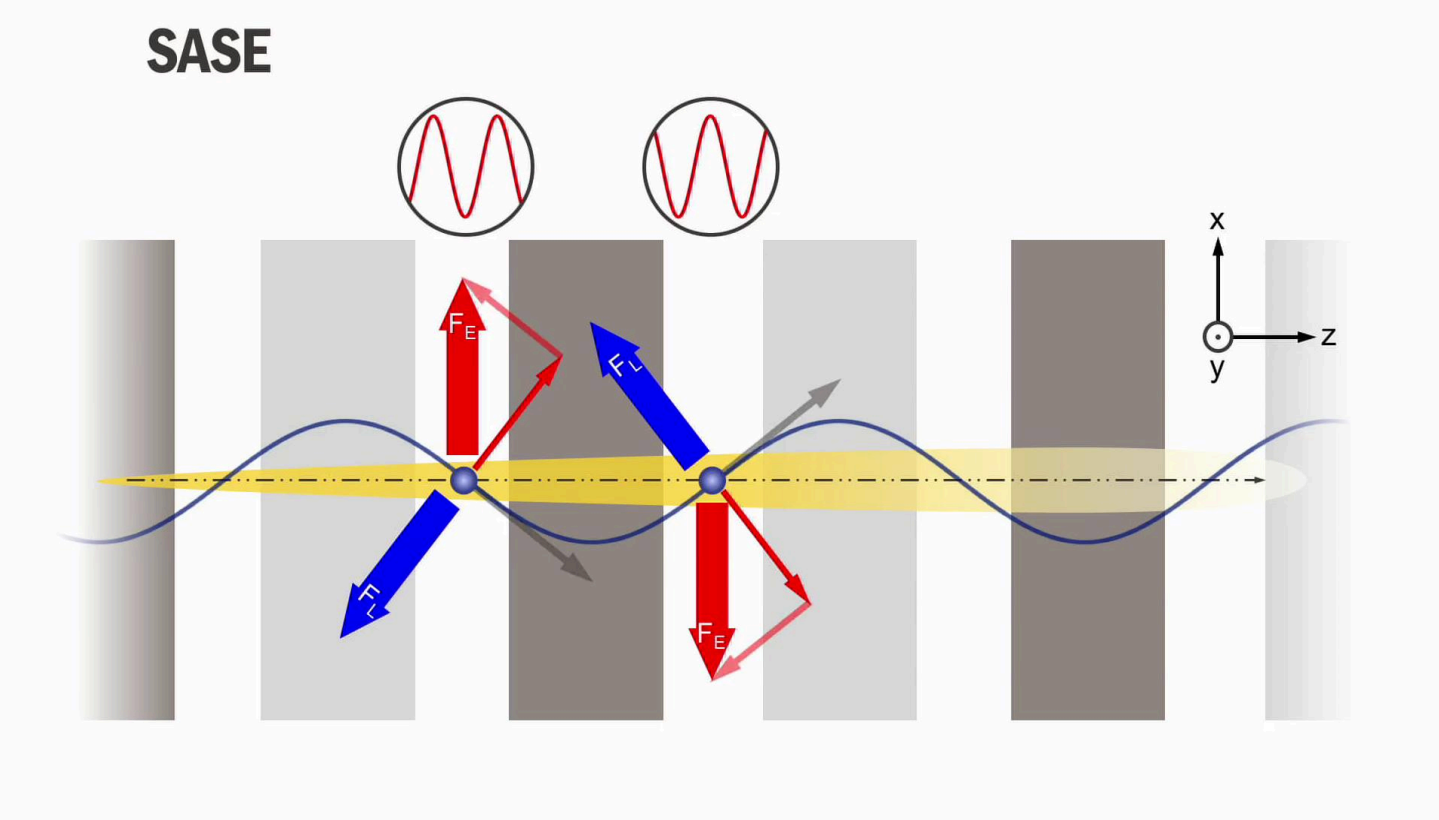
- Electrons move forward by one undulator period λ_u
- EM radiation moves forward by odd integer number of wavelengths relative to electrons

Now let's mix things up a little. Instead of the electron moving precisely along the same axis as the electromagnetic field, we send it through an array of undulator magnets. Remember that for constructive interference to be met in an undulator as the electrons move forward by one undulator period, Lamda U, the electromagnetic radiation emitted at the beginning of the period must have advanced by an odd integer number of wavelengths at the end of the period. As such, the electrons feel the same electric and magnetic fields at each end of a period and the opposite set of fields exactly in between.

Notes

Summary

2m 58s

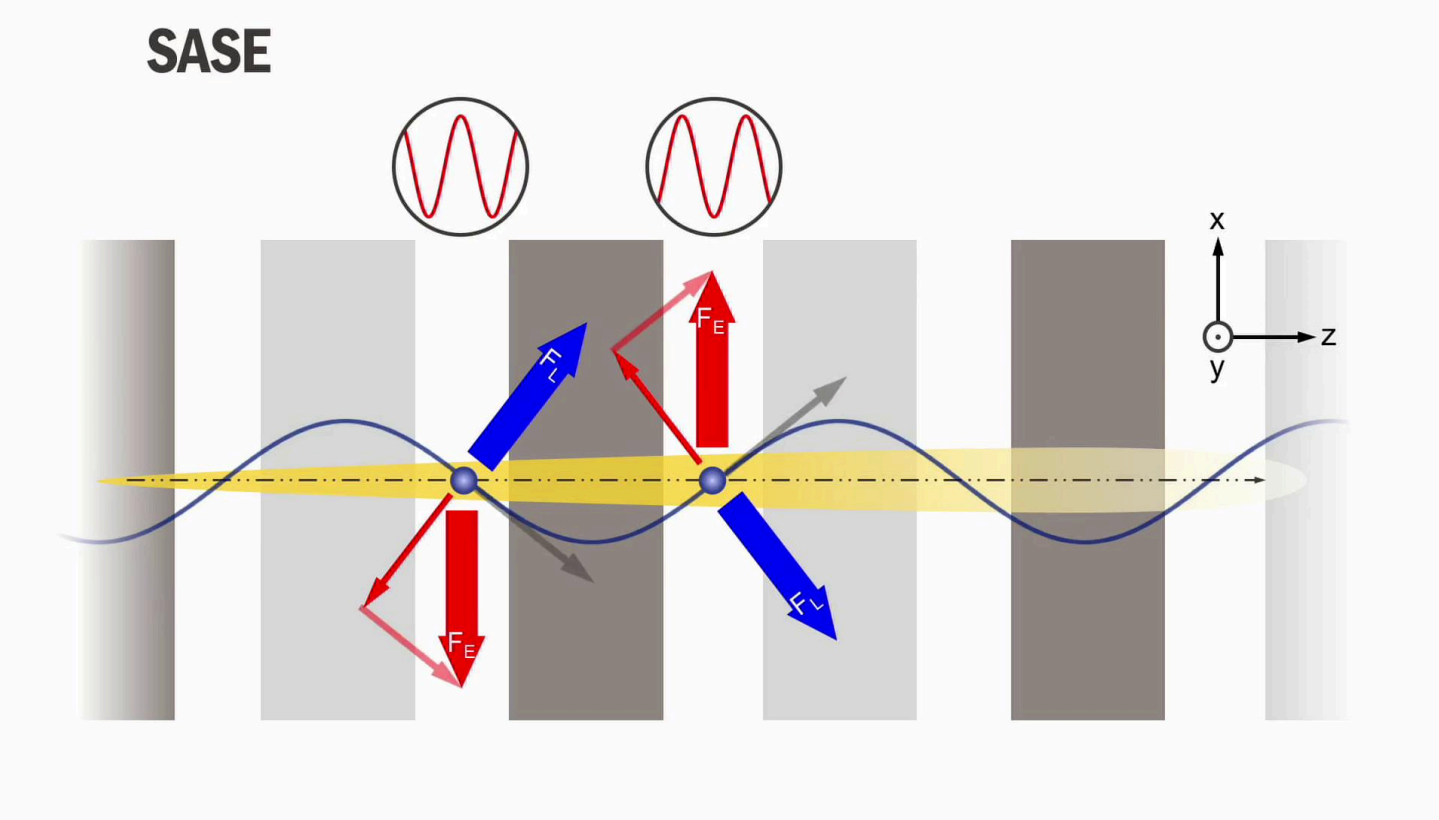


Now let's see what forces act on the electrons. We concentrate on the positions along the electrons trajectory, along the undulator, where their angle to the central axis is maximal. This lies exactly in between the magnet poles. We have two situations. One where the electron is moving at a positive angle relative to the axis, and another, half an undulated period later, where the angle is negative as shown here. In these two situations, the electromagnetic fields are reversed with respect to each other. In the example shown here, the electric field of the left upstream electron points in the negative X direction and the force in the positive X direction. While for the downstream electron, the opposite is true. The Lorentz forces act in the direction shown by the blue arrows, in directions that point marginally upstream. We now decompose the electric force into a component antiparallel to the Lorentz force and another perpendicular to it. Note that the perpendicular component acts in the opposite direction of the direction of motion of the electron. The net effect is that the energy is transferred from the electron to the electric field. In other words, the electron is decelerated.

Notes

Summary

3m 47s



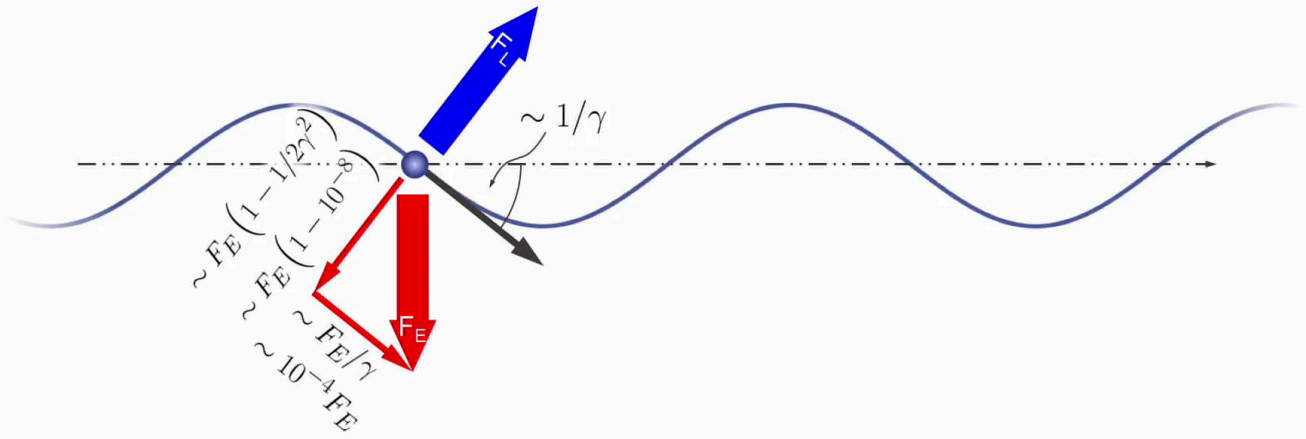
Now let's consider the case where the electromagnetic field at the upstream electron is exactly Pi out of phase compared to the example just shown before. This phase shift of Pi will obviously apply to the downstream electron as well. If we now draw the electric and Lorentz forces in this new situation, we see that their directions have flipped. The upstream electron experiences an electric force in the negative direction, and the downstream electron one in the positive direction. The Lorentz forces for both electrons now point marginally forwards. Again, we break down the electric forces into perpendicular and antiparallel components relative to the Lorentz forces. This time we see that the perpendicular parts act in the positive direction relative to the electrons motion. Energy is in this case transferred from the electric field to the electrons.

Notes

Summary

5m 27s

SASE



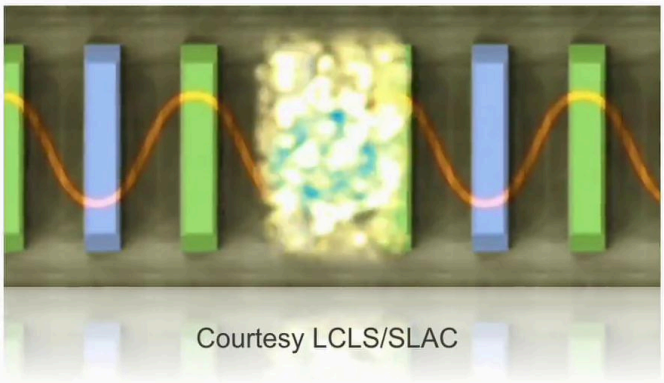
Now what are the magnitude of these components of the electric force? The maximum angle subtended by the direction of motion and the central axis is, you may remember, one upon Gamma. Hence the component of the electric force perpendicular to the Lorentz force is FE times the sign of one upon Gamma. Which for small angles is very precisely approximated by FE divided by Gamma, or about one ten thousandth of FE. The antiparallel component is FE multiplied by cosine one upon Gamma, which again for small angles, is excellently approximated by FE multiplied by one minus two upon Gamma squared, which is exactly equal but opposite to the Lorentz forces.

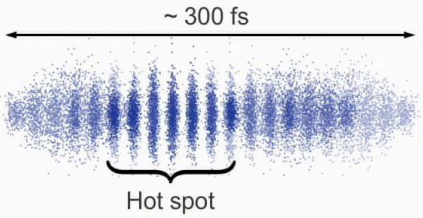
Notes

Summary

6m 41s

SASE microbunching





- Electrons in undulator B-field experience unbalanced forces from the E- and B-fields of the radiation they produce
  - Some electrons accelerated
  - Some electrons decelerated
  - Leads to microbunching with femtosecond durations

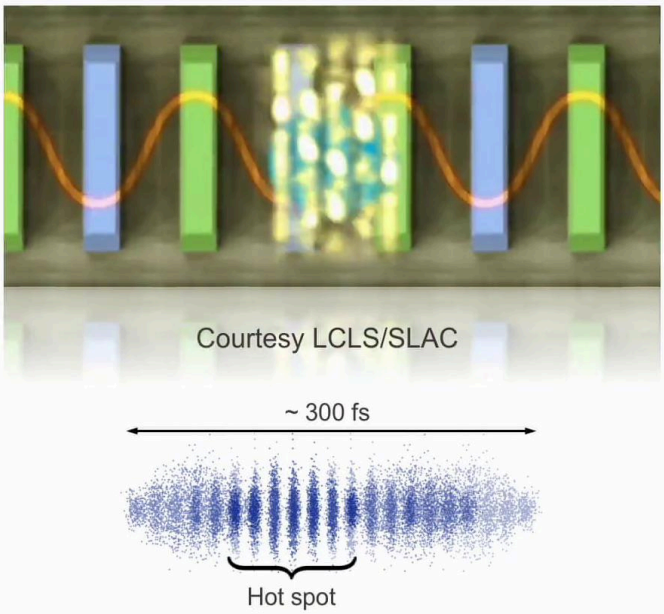
So in summary, the electrons in the undulate magnetic field experience unbalanced forces from the electromagnetic field that they themselves generate and are bathed in. According to the relative phase of the electromagnetic field, with the position of the electrons within an undulated period, some of them will be accelerated, some of them will be decelerated. This in turn leads to micro bunching. A bunch of electrons that entered the long undulator as a continuous cloud of duration of about half a picosecond and length of approximately 100 microns emerges at the far end as a tray of up to 100 million micro bunches for hard X-rays separated by the wavelength of light they generate. This is SASE. Note that SASE does not occur perfectly along the entire bunch length of 100 microns but instead occurs only in the most intense portions of the bunch. As such, SASE is a stochastic process. The number of involved micro bunches in these hot spots determines the X-ray pulse duration. This can lie anywhere between a few tens of femtoseconds or hot spots that stretch as much as approximately 10 percent of the original electron bunch and contain 100,000 micro bunches or 10 to the nine electrons, and under one femtosecond containing maybe only approximately a thousand bunches and 10 to the seven electrons.

Notes

Summary

7m 37s

SASE microbunching



- Electrons in undulator B-field experience unbalanced forces from the E- and B-fields of the radiation they produce
  - Some electrons accelerated
  - Some electrons decelerated
  - Leads to microbunching with femtosecond durations
- The more tightly the electrons are microbunched, the stronger the EM-field
  - ⇒ stronger forces
  - ⇒ increased microbunching
  - ⇒ runaway effect: SASE

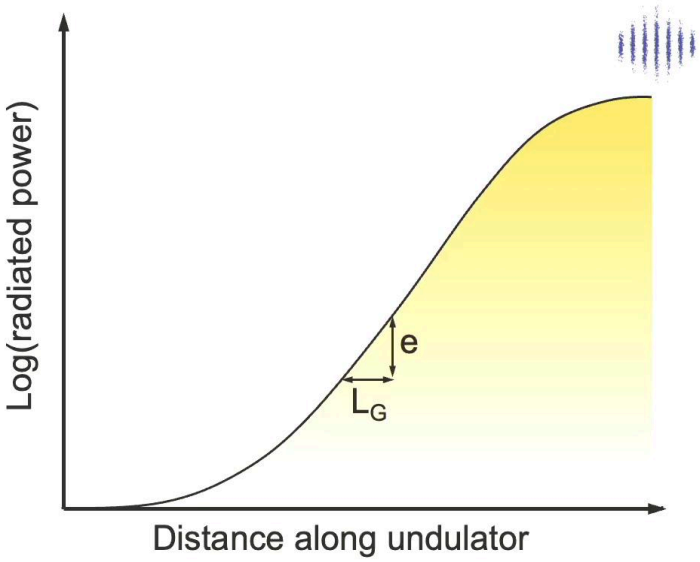
SASE is a runaway effect. The more the electrons bunch together into micro bunches, the stronger is the radiation they produce, leading to stronger associated forces on those electrons and still higher micro bunching.

Notes

Summary

9m 20s

SASE, the gain length, and the Pierce parameter



- XFEL gain length

$$L_G = \frac{\lambda_u}{4\pi\sqrt{3}\rho_{\mathrm{FEL}}}$$

~ 2 - 5 m

- Pierce (or FEL) parameter

$$\rho_{\mathrm{FEL}} = \left[\frac{\lambda_u^2 r_0 n_e K^2}{32\pi\gamma^3}\right]^{1/3}$$

~ 5 x 10^{-4}

$$\rho_{\mathrm{FEL}} = \frac{\Delta\omega}{\omega} \qquad \rho_{\mathrm{FEL}} = \frac{P_{\mathrm{ph}}}{P_e}$$

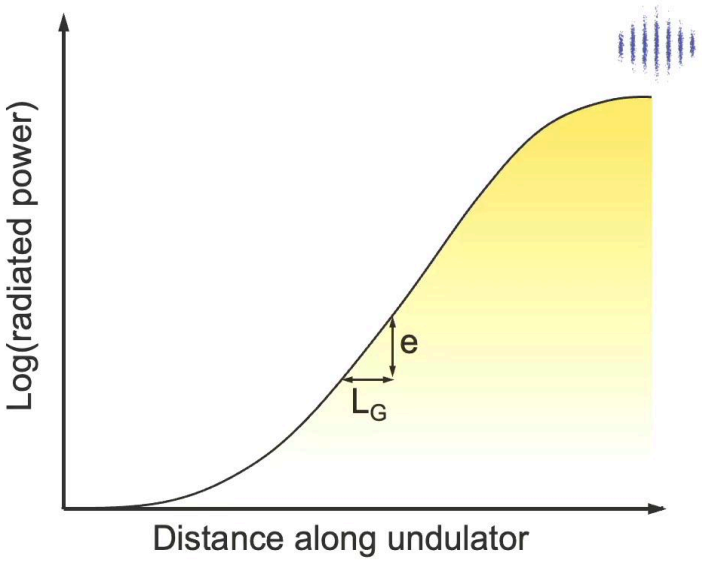
The radiated power of the micro bunches increases quasi exponentially as they travel along the undulator, reaching saturation shortly before the end. Now in the main exponential regime, the gain length, LG, is defined as the distance along the undulator required for the power to increase by a factor E, or about 2.72. It is given by the equation LG is equal to Lambda U divided by four Pi, root three Rho FEL, or Lambda U is equal to 21.77 Rho FEL. Now Rho FEL is called the Pierce or FEL parameter and is given by the equation shown here. It expresses the fraction of the electron power that is converted into photonics power and also the instantaneous relative bandwidth of the radiation. Typical electron densities in the compressed electron bunch entering the undulator are of the order of around 10 to the four per cubic micron. That is around 10 to the nine electrons in a volume of the order of 10 to the five cubic microns. The undulators have periods of 15 to 30 millimetres and K values of approximately two to three. And the linux produce electron energies of the order of ten giga electron volts.

Notes

Summary

9m 38s

SASE, the gain length, and the Pierce parameter



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~ 5 x 10^{-4}

$$\rho_{\mathrm{FEL}} = \frac{\Delta\omega}{\omega} \qquad \rho_{\mathrm{FEL}} = \frac{P_{\mathrm{ph}}}{P_e}$$

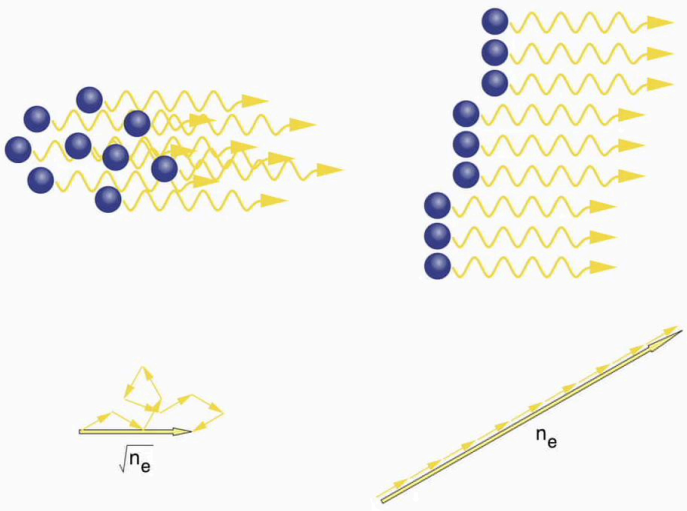
Now, because of the cube root dependency of Rho FEL on all the design variables, with the exception of the electron energy, its spread is fairly narrow across all facilities designed or commissioned to date, and lies it around five times 10 to the minus four. Returning to the gain length. If we plug in typical values for Lambda U and Rho FEL, we obtain gain lengths of the order of two to five metres. Because saturation occurs after a SASE gain of approximately five orders of magnitude, the total undulator length should exceed LG by a factor of around 11 or more. The reason for this is that 10 to the five is equal to E to the eleven, approximately. XFEL undulators are therefore measured in several tens, or even hundreds of metres.

Notes

Summary

11m 20s

Why SASE microbunches are so intense



Noncoherent bunch Coherent bunch

- Relative phases between photons emitted by noncoherent bunch is random
  - E-field amplitude: vector sum $\propto n_e^{1/2}$
  - Light intensity $\propto E^2 \propto n_e$
- Photons in microbunches all in phase
  - E-field amplitude $\propto n_e$
  - Light intensity $\propto E^2 \propto n_e^2$
- $n_e \sim 10^9 - 10^{10}$

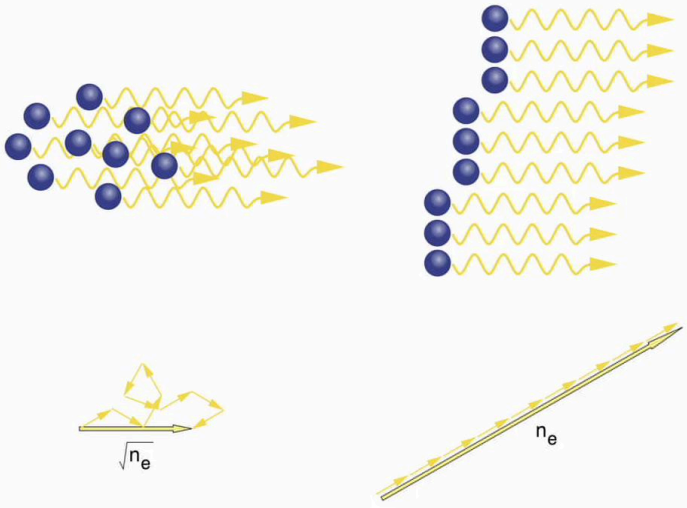
As we have already seen in the previous video, the peak power output of XFELs exceeds that of synchrotrons by some 10 orders of magnitude or the order of a factor of 10 billion. The reason for this also lies in the SASE process. The electrons are rearranged into micro bunches with separations equal to the wavelength of radiation they produce. This means that the electromagnetic fields or wave trains generated by the electrons within a hot spot are all in phase. The electric field amplitudes therefore add up linearly or in a straight line, in an Argand diagram. The radiation is said to be coherent. The intensity is proportional to the square of the amplitude and is therefore proportional to the square of NE, the number of electrons within the hot spot. In contrast, in the case of synchrotron radiation, emission of radiation is entirely stochastic. It does not depend on the position of the radiating electron relative to any of the other electrons. The phase relationship between the irradiated electromagnetic fields is therefore also completely random. In a phase diagram or Argand diagram, this leads to a drunkard's walk vector edition of the electromagnetic field vectors, for which the sum length is proportional to the square root of the number of contributing components.

Notes

Summary

12m 20s

Why SASE microbunches are so intense



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- Photons in microbunches all in phase
  - E-field amplitude $\propto n_e$
  - Light intensity $\propto E^2 \propto n_e^2$
- $n_e \sim 10^9 - 10^{10}$

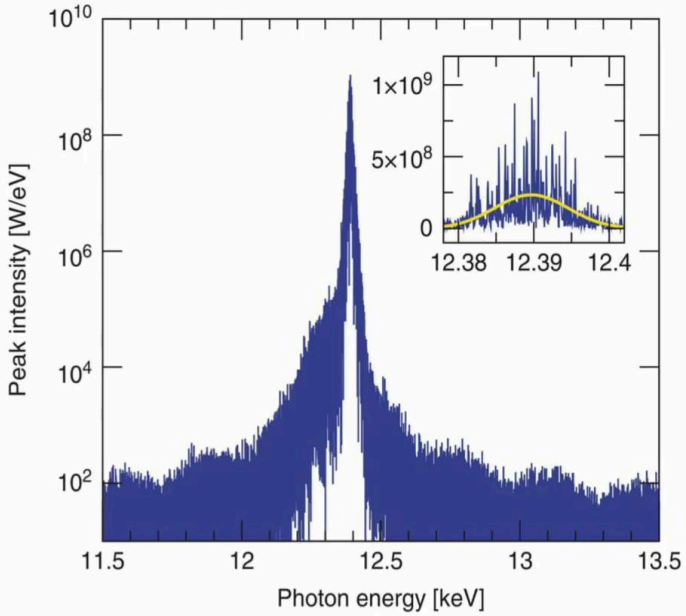
In other words, the sumed amplitude is proportional to the square root of NE, and the intensity is thus proportional directly to NE. The difference, therefore, between the intensity of a synchrotron and XFEL beam is a factor approximately equal to NE, which is of the order of one to 10 billion.

Notes

Summary

14m 01s

SASE energy spectra



- Any given pulse
  - Intrinsic bandwidth $\Delta\nu/\nu \sim 5 \times 10^{-4}$
  - e.g. @ 10 keV, $\Delta E \sim 5$ eV
  - Central energy jitter ~ 40 eV
- SASE radiation much broader than monochromatized synchrotron radiation
  - Many experiments acceptable
  - Some experiments require narrower bandwidths
  - How?

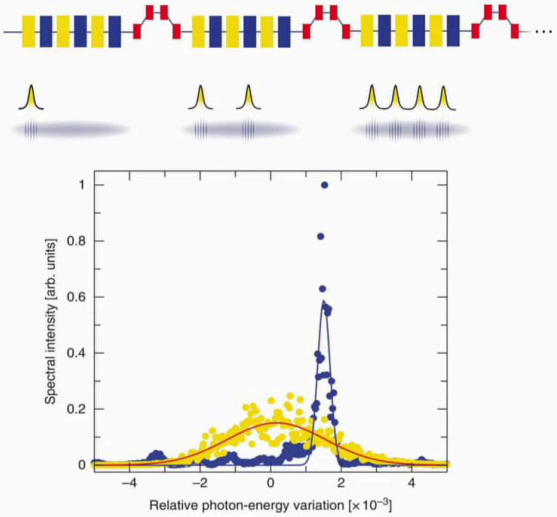
SASE radiation exhibits an instantaneous bandwidth, that is, the bandwidth of any one FEL pulse of the order of five electron volts for 10 kilo electron volts photons, according to the Pierce parameter definition. This is well over an order of magnitude larger than the transmitted bandwidth after monochromatization with a diamond 004 single crystal. In addition, the position of the centre of the SASE spectrum jitters by a few tens of electron volts from pulse to pulse. It is the stochastic nature of the initial spontaneous generation of SASE radiation, which is responsible for this fairly large bandwidth. Now, this is acceptable for many types of experiment, such as in serial femtosecond crystallography, but might be problematic in others, especially in spectroscopy. The question is, therefore, how can one generate narrower peaks?

Notes

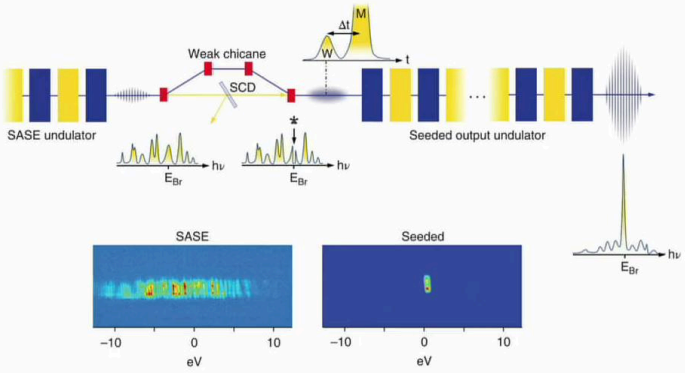
Summary

14m 26s

Seeding and monochromatizing SASE radiation



High-brightness SASE



Self-seeding

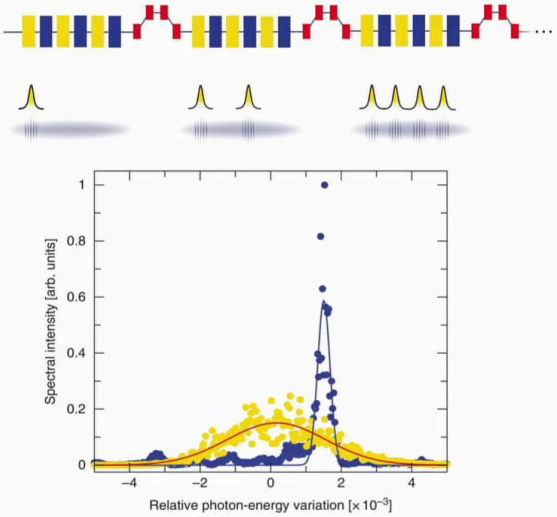
Here we show just two possibilities. On the left is the set-up used in so called high brightness SASE. In HB SASE, short undulator sections, each typically two metres long, are separated by weak chicanes consisting of four dipole pairs. The XFEL field generated in the first undulator thus induces SASE in a new region of the original electron bunch further downstream in the second undulator. Importantly, the separated SASE regions are in phase with each other. Both of these can now induce new SASE regions in the next undulator section. In the configuration shown here, each successive delay induced by the chicane is half as long as the previous, causing the SASE regions to fill the gaps as it were. Self-seeding of hard X-rays is now routine at XFELs using an intra-undulated diamond single crystal. This works best for short pulse lengths below approximately 20 femtoseconds for reasons that will become immediately apparent. In this scheme, a diamond single crystal acts as a Bragg transmission monochromator. At the Bragg diffraction condition, the temporal form of the transmitted beam has delayed long monochromatic wake pulse with a width inversely proportional to the bandwidth of the extinction line in the transmission spectrum.

Notes

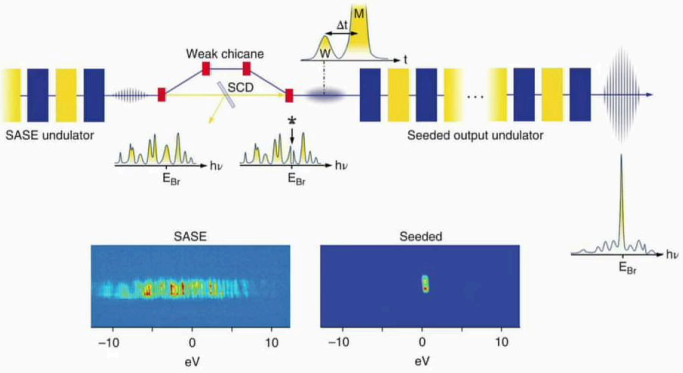
Summary

15m 33s

Seeding and monochromatizing SASE radiation



High-brightness SASE



Self-seeding

This is the feature highlighted by this small asterisk. This radiation, delayed by about 20 femtosecond compared to the diffracted beam is made to physically overlap with the electron pulse in the second undulator array and thus provide the monochromatic seeding. The required overlap is achieved by the weak form magnet chicane that is also used to provide space for the diamond crystal and wash out the incipient SASE microbunching produced by the SASE undulator.

Notes

Summary

17m 15s

In the next video, we'll look at the selection of scientific highlights that have emerged from XFELs in the decade or so since they first appeared.

Notes

Summary

17m 49s